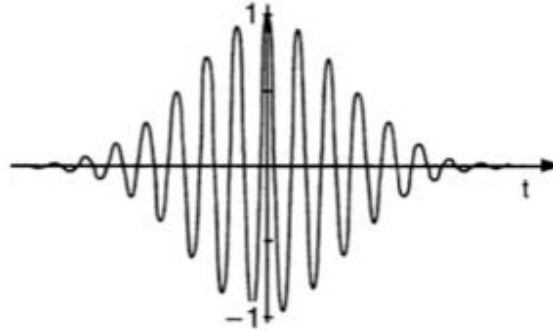


Laser: Theory and Modern Applications

ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE (EPFL)

Exercise Sheet 10: Pulse characterization

10.1 Pulse characterization The pulse shown above as Gaussian envelope and carrier frequency ω . Its electric field is given by:

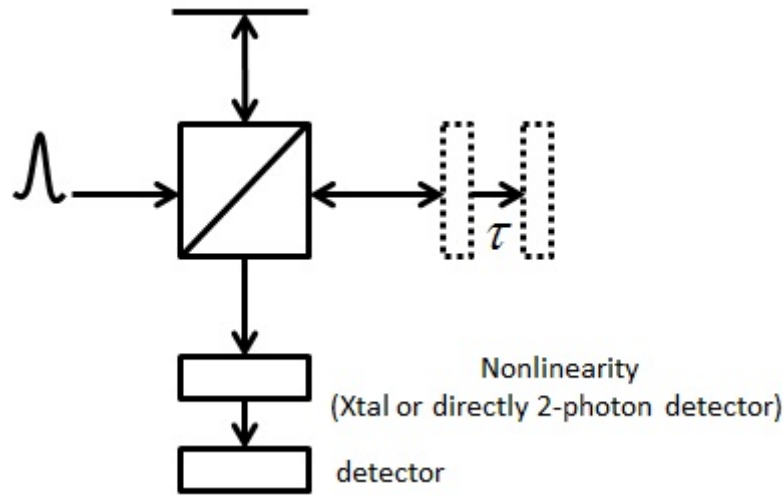
$$E(t) = e^{-0.5(\frac{t}{t_0})^2} e^{j\omega t}$$

1. Calculate the temporal FWHM of the pulse intensity.
2. Calculate the Fourier Transform of $E(t)$ and the FWHM bandwidth.
3. To experimentally measure the duration of this pulse, the experimenter wishes to use a second order autocorrelator of the non-colinear type. Calculate the expected signal coming out of this autocorrelator. (use the field $E(t)$ above for the pulse).
4. Calculate the FWHM t'_0 of the function derived in 3. This is the pulse width measured by the experimenter. What factor does this value need to be multiplied by to recover the original pulse width.
5. A particular pulse shape that often appear in various mode-locked system is called "soliton". Soliton represents a self-maintained waveform that relies on the balance between the dispersion and nonlinear effects in the medium, where it propagates. This allows him to maintain the same shape over extremely long distances. Assuming that the soliton pulse has a form of $\text{sech}^2(t)$ calculate it's time-bandwidth product.

10.2 Compute the signal from a second order autocorrelator, i.e. compute the interferometric second order intensity.

$$I_2(\tau) = \int_{-\infty}^{\infty} |E(t) + E(t - \tau)|^2 dt$$

Take $E(t) = E_0(t) \cdot e^{j(\omega t + \phi(t))}$



1. Show that

$$I_2(\tau) = \int_{-\infty}^{\infty} [2E_0^4(t) + 4E_0^2(t)E_0^2(t-\tau) + 4E_0(t)E_0(t-\tau)[E_0^2(t) + E_0^2(t-\tau)] \cos(\omega\tau + \phi(t) - \phi(t-\tau)) + 2E_0^2(t)E_0^2(t-\tau) \cos(2\omega\tau + 2\phi(t) - 2\phi(t-\tau))] dt$$

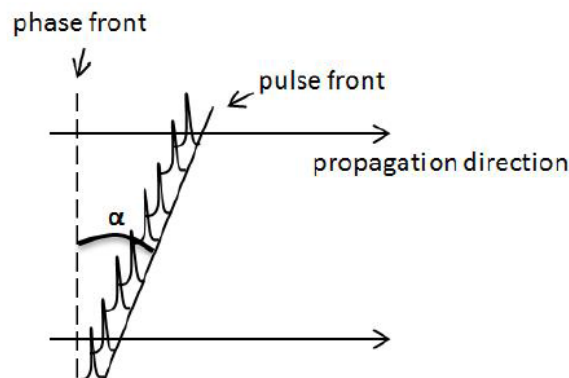
2. Compute $I_2(\tau)$ for the case when the average sweep time τ is much faster than the detector time response.
3. Compute the ratio between the background ($\tau \rightarrow \infty$) and maximum signal ($\tau = 0$) of the interferometric autocorrelation in 11.2.1 and the intensity autocorrelation function in 11.2.2.

10.3 Let's assume, we have a function $f(t)$ (unknown) that is measured with a field correlator with a known gate function $I(t)$. ($I(t)$ is a pulse of smaller duration than $f(t)$).

How can the function $f(t)$ be calculated?

Hint: Write the field correlation function and use a property of the Fourier transform.

10.4 In the case of a single shot autocorrelator, one of the beams has a pulse front field with a tilting angle α as shown in the figure below:



Such a pulse front tilt can be generated simply by refracting at an interface:

constant and is equal to 18 ps/(km · nm).

1. Consider the pulse propagation equation in a linear, dispersive medium: $i \frac{\partial E(z,t)}{\partial z} = \frac{\beta_2}{2} \frac{\partial^2 E(z,t)}{\partial t^2}$, where $E(z,t)$ is the pulse amplitude, and β_2 is the group velocity dispersion (GVD) parameter. Solve this equation using the Fourier-transform method to show that the spectrum of any optical pulse will be changing according to:

$$\tilde{E}(\omega, z) = \tilde{E}(\omega, 0) e^{i \frac{\beta_2}{2} \omega^2 z}$$

where $\tilde{E}(\omega, 0)$ is the Fourier transform of the pulse at $z = 0$.

2. Calculate $\tilde{E}(\omega, 0)$ for the Gaussian pulse having the form of $E(t, 0) = e^{(-\frac{t}{2t_0})^2}$, and use the inverse Fourier transform to show that the pulse profile in the time domain will have the following form:

$$E(z, t) = \frac{t_0}{\sqrt{(t_0^2 - i\beta_2 z)}} \exp\left(\frac{t_0^2}{2(t_0^2 - i\beta_2 z)}\right)$$

3. Knowing the value of the fiber dispersion and initial pulse durations ($t_0 = 25$ fs and $t_0 = 100$ fs), calculate their broadening after the fiber propagation (fiber length $L = 1$ meter).